SOLUTION

CALCULUS II **QUIZ VI** MATH-UA.0122-007

Name:

Write your solutions in steps. You need to provide explanations to your answer.

1. (4 points) Determine if the following sequences converges or diverges. Find the limit if it converges.
   (i). $\{\frac{2n+e^n}{n^2}\}$.
   (ii). $\{\ln(1+\frac{1}{n})\}$
2. (6 points) Determine if the following series converges or diverges.
   (i). $\sum \frac{1}{1-2n}$
   (ii). $\sum \frac{(-1)^n}{e^{2n+1}}$
   (iii). $\sum \frac{\cos 3n}{n^2}$

1(i). $\lim_{x\to+\infty} \frac{2x+e^x}{x^2} = \lim_{x\to+\infty} \frac{2+e^x}{2x}$

$= \lim_{x\to+\infty} \frac{e^x}{2}$

$= +\infty$

So $\{\frac{2n+e^n}{n^2}\}$ diverges

1(ii). $\lim_{n\to\infty}(1+\frac{1}{n}) = 1+\lim_{n\to\infty}\frac{1}{n} = 1$

and $f(x)=\ln x$ is continuous at $x=1$.

So $\lim_{n\to\infty}\ln(1+\frac{1}{n}) = \ln(\lim_{n\to\infty}(1+\frac{1}{n})) = \ln 1$

$=0$

The sequence converges to 0.

2(i). $\lim_{n\to\infty} \frac{\frac{1}{1-2n}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{1-2n} = -\frac{1}{2} \neq 0$.

and $\sum\frac{1}{n}$ diverges ~~converges~~.

So by Limiting Convergence Test.

$\sum\frac{1}{1-2n}$ diverges.

2(ii). $\{\frac{1}{e^{2n+1}}\}$ is a positive decreasing sequence with $\lim_{n\to\infty}\frac{1}{e^{2n+1}}=0$, so by Alternating Convergence Test.

$\sum\frac{(-1)^n}{e^{2n+1}}$ Converges.

2(iii).

$0 \le |\frac{\cos 3n}{n^2}| \le \frac{1}{n^2}$

$\sum\frac{1}{n^2}$ Converges, by Comparison Test. $\sum|\frac{\cos 3n}{n^2}|$ Converges, then by Absolute Convergent Test

$\sum\frac{\cos 3n}{n^2}$ Converges.